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Auctions

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Course Registration Auctions

Auctions are a family of market mechanisms which allow the optimal exchange of goods in settings of asymmetric information, such as in situations where the seller does not know how the buyers will value the item for sale, or potential buyers all value the item differently. One "market" which exhibits these asymmetries is course registration at colleges and universities. Many institutions, particularly business schools, use an auction to allocate seats in courses, but the format and rules vary widely across these implementations. This project focuses on an analysis and comparison of some of these auctions and concludes with the design of a course registration auction to be theoretically implemented in place of Colby College's current registration system.

The field of mechanism design is concerned with creating the rules and procedures for how individuals interact in a strategic setting with some specific goal in mind, such as eliciting information, forcing truthful behavior, or profit maximization. The areas of auction theory and mechanism design are closely related, as auctions are mechanisms which are used to buy and sell items. For example, the Vickrey Auction (second price, sealed bid) is a mechanism which can be used to both allocate an item to the individual who values it the most while simultaneously learning every bidder's true valuation of the item. Implicit in this example is the asymmetry of information between the buyer and seller. If the seller knew *a priori* which individual had the

highest value for the item and what that value was, there would be no need for the auction as the two could engage in a direct transaction. The auction bridges this gap in the information and allows the seller to learn this information from the buyer. College course registration is a prime example of a situation where the "seller" of the goods (the registrar's office) knows very little about the preferences of the individual "buyers" of the goods (the students). This suggests that an auction mechanism may be appropriate for allocating seats in courses to students.

In considering course allocation mechanisms, there are numerous metrics, factors, and constraints to keep in mind. Some mechanisms may perform well in some of these at the expense of performing poorly in others, i.e. a system which allocates all seats to students purely at random is certainly fast to implement and easy to understand, but it does a very poor job of respecting students preferences. Graves, *et.al.* (1993) identify three important design-side factors in establishing the course registration auction at the University of Chicago's Graduate School of Business (GSB), the first two of which are fundamental across all course registration auctions, and the third concerns the specific implementation of the GSB's auction. The first factor is that the mechanism must allow students to express preferences for courses. Because preferences are highly individual and can depend on a variety of course aspects from subject material to meeting time to the teacher to the other students in the course, students must be able to express these preferences in the registration process. The second factor is that the mechanism must respect course seat capacities as a binding constraint. This is relatively straightforward, as the number of seats in a classroom cannot be infinite, so there must be some expressible upper limit on the maximum number of students allowed in a course. The third factor, which depends more heavily on the implementation of the auction is that students must be able to express

changes in their preferences during the process. At a minimum, this is accomplished through a first-come-first-served open add/drop period following the auction, as it only needs to be assured that a student who decides after winning a course that she no longer wants her seat is able to drop that course and a student who decides she wants a seat in a course she did not bid on can pick up an empty seat or add herself to a waitlist.

Beyond these three factors which should be inherent to the auction mechanism, there is a wide variety across the implementation details for course registration auctions in decisions about the auction being single- or multi-round, how currency endowments are given to students, bid prioritization, and the rules of the add/drop phase. This paper will analyze a sample of course registration auctions which vary in all of these factors.

The first, and perhaps the most straightforward example is the one in use at Colorado College, a small liberal arts institution in Colorado Springs, CO. Benson, *et.al.* (2013) provide a structural outline and econometric analysis of this auction mechanism, including some of the idiosyncrasies of the process. The auction is one of the simpler ones in that it involves a non-transferrable[1], non-bankable[2], and non-refundable[3] point endowment for each student. Colorado College operates on a "block" schedule rather than the more traditional semester system; each academic year, students enroll in at most one course in each of eight three-and-a-half week long blocks. At the time of registration, each student is given an endowment of eighty points (first years are given sixty points, but their first two blocks are predetermined at matriculation). Students then submit bids for their choice of courses. Because points are not bankable and bidding occurs in a single round, students are advised to spend all of

[1] Students cannot directly transfer courses or points to one another
[2] A balance of unspent points does not carry over to the next registration period
[3] Students cannot receive back their spent points by dropping the course after winning it at auction

their points. Students are added to courses in descending order of bids, which effectively makes this a first price sealed bid auction. For courses which receive more bids than there are seats, a wait list is maintained in descending order of bids. Following the resolution of the auction, there is an open add/drop period where students can freely drop won seats and add courses with open seats.

Given the parameters of the auction, it would be nice if each student's bid profile truly reflected her preferences; that two courses valued equally were given equal bids and courses with no value were given bids of zero. Unfortunately, this is not the case and the system allows for and encourages strategic deviation from this weighted preference paradigm. The registrar's office provides the year's complete schedule, so students have access to total information about how many times a course will be offered during an academic year as well as a document with the bids and results from the previous year, both of which should inform bidding strategies for students. Benson, *et.al.* found in their analysis that some departments had average bids fifteen points higher than others, which suggests that competition for courses in these departments is relatively high and the demand for seats exceeds the supply. Therefore, while the endowment and bidding procedure should theoretically produce a expression of weighted ranked preferences from each student, the system does not encourage the bidders to necessarily behave truthfully, especially in registration for these popular courses. For example a student who values a popular biology class equally with an unpopular theater class is not incentivized to bid equally for the courses. Rather, since she is confident that the theater class will not reach capacity, she is better off bidding high for the biology class and low for the theater class because the additional

information given from the bid history allows her to formulate a bid profile that maximizes her expected probability of gaining a seat in both courses.

On top of this, while the system appears equitable in that all students begin with the same eighty point endowment and access to the historical information, one quirk of the system is that every student receives the full endowment, regardless of how many blocks she actually intends to take a course. For example, a student spending a semester abroad is effectively twice as wealthy as a student spending the full year on campus. While this can be viewed as a positive consequence in that it gives some priority to students who do not have the option to take a course in a later term, it could also be considered a negative effect in that giving some students more wealth distorts the market and erodes the equity of the system.

As noted, course registration auctions are frequently used in business schools; Colorado College is perhaps one of the only primarily undergraduate institutions which use one to allocate seats in courses. One possible explanation for their use in elite business schools in particular is that students in these schools are more likely to be able to understand and confidently participate in such a market structure and a relative homogeneity among the students in that each student likely wants to take some mixture of popular and unpopular courses, whereas students at an undergraduate liberal arts school may be in a popular major and need to take many popular courses or an unpopular major and not need to worry about not getting a seat in the course. The next three auctions discussed are in use at business schools and are presented in roughly increasing order of complexity.

Northwestern University's Kellogg School of Business' auction shares some features with the Colorado College one. Students, with the exception of their first term enrolled,

participate in a cycle of two-round auctions to register for courses. The process occurs for each academic quarter and points not used in the a quarter can be carried over to the auction for courses in the next quarter. Like Colorado College, each student begins with a non-transferrable, non-bankable endowment with which to bid on a full year's course load. Unlike Colorado College, however, these points are refundable after the first round in order to facilitate the second round of bidding. Students submit bids in the first round, after which seats in courses are awarded to the highest bidders at the price of the lowest successful bid, called the "clearing price". Students who bid above this price are refunded their excess. Students can also specify bids for alternate sections of a course which, if they are not allotted a seat in their first choice, can get a seat in another section of that same course. If this occurs, the student is not charged any points, as the availability of seats implies the clearing price for that section was zero. Following the resolution of this first round, students can drop classes they have a seat in for the full amount they paid. This step likely exists because the system allows students to bid on courses that meet concurrently, and a student who wins both of these seats must drop at least one, and the full-refund drop allows that to occur with no penalty. Subsequently, there is a second and third round of bidding. The second round is for bids on seats in courses that did not fill after the first round of bids. This auction is independent of the first, as only the empty seats are considered, and a round two bidder cannot displace a round one winner, even if he bids higher than the round one clearing price. Following the resolution of this round, students can use their remaining points to bid on positions in course wait lists, and are ordered by descending bids. This differs from Colorado College's system where the wait list is automatically formed by failed bidders. Following waitlist bidding, students can freely add courses with empty seats and

drop courses they currently have. These add/drop rounds progress with students receiving 75 percent, 50 percent, 25 percent, and no refund for dropping courses won at auction. These refunds carry into their balance for the start of the first round of bidding for the next term.

There are a few key features to note when considering strategic behavior in this auction. The first is that while points cannot be carried from year to year, they do carry from term to term with a new endowment also given at the start of first round bidding, although the amount of the endowment depends on each student's status; part-time students and evening students receive less than full-time students. This suggests that students will be richer in later rounds than in earlier rounds and that there is no incentive to not spend all of one's points when bidding in the final term. Both of these imply bids should be higher in courses during later terms. Additionally, unlike the Colorado College auction, students receive points for each term rather than for the whole year, so there is no wealth bias for students not taking courses during one or more terms. Finally, because students pay the clearing price and not their bid for seats in a class, the auctions should be theoretically closer to a second price auction and it should result that students bid their actual valuation for courses and bid data should more closely reflect individuals preferences for courses, appropriately discounted for the potential to save points for courses in future terms. Unfortunately, without access to this bid data, only theoretical analysis is possible.

The University of Chicago Booth School of Business' auction is just one step up in complexity from Kellogg's. The formats of the auctions are nearly identical. Booth uses five rounds of bidding followed by full-refund drops where Kellogg uses two, and Booth finalizes wait lists using bids from the fifth round instead of having a separate round for those spaces.

The pricing rule is identical, where the winners are those with the highest bids and they pay a clearing price equal to the lowest successful bid and uses a similar process for alternate section registration. Where Booth's system differs is in point accrual and endowment. Unlike the previous two auctions where endowments were given at the start of registration periods, at Booth, a one-time endowment of 8000 points is given at the start of a student's first auction process and unspent balances not only carry over from term to term but from year to year. Additionally, instead of receiving an endowment at the start of each registration period, students add 2000 points to their balance upon the successful completion of each course. This means that it should be expected that on average, students become wealthier from term to term, but unlike in the Kellogg auction where additional endowment supplements the student's balance, there is no guarantee that each student will be richer when moving to successive rounds.

Discussing optimal strategy in this auction format is difficult due to the income system. A student may be made best off by taking inexpensive courses at the start of her program to bank points to spend on expensive courses in later terms or years. On the other hand, a less patient student might try to take expensive courses early and maintain a low balance. A student could also use a mix of the two strategies which might involve taking some inexpensive courses to maintain a point reserve while periodically taking expensive courses throughout her studies. The advantage to the Booth system is that it is highly equitable and theoretically allows all students the same opportunity to access courses in high demand. If a student cannot get a seat in an expensive course in a given term, she will have the opportunity to bid for it again in a later term using her larger balance. One possible strategic issue stems from granting points for completion of courses. While this mechanism does successfully reward students who complete courses and

punish students who do not, it creates an incentive for taking additional courses, which could have the adverse effect of creating extra demand for courses therefore driving up prices. That is to say, if a typical student takes three courses in a term, there may be a group of students who are willing to add a fourth course if the price is sufficiently below 2000 points. These students would bid a low amount, say 500, for a course which would otherwise have an empty seat in the absence of these incentives, thus forcing those who actually want to take this class to pay 500 instead of zero to take the course.

The Kellogg and Booth models are fairly similar to a handful of other course registration auctions. The most notable deviation from these structures is the course auction formerly in place[4] at the University of Pennsylvania's Wharton School. The first portion is structurally identical to Booth's auction. Students are given a one-time endowment of 5000 points at enrollment and receive 1000 additional points for each course completed and bidding takes place over several rounds, with the exception that students cannot receive refunds for dropping a course during the auction portion. Following the completion of the auction, the process enters an open market phase where over the course of several rounds, students can buy and sell courses from each other. Students wanting to sell a seat offer the seat with an ask price and those wanting to buy a seat submit a bid. This spread is then used to determine a clearing price, much like a stock market, at which the transactions occurred for sellers asking less and buyers bidding more than this value. Given this component of the market, it is difficult to describe any ideal strategic behavior. Students may try to buy up seats in early auction rounds to sell later at a

[4] Wharton has since changed to a system called Course Match, where students express relative preference weightings across all courses and an algorithm performs scheduling against a student's preferences and a predetermined budget. While this system is really cool and probably more fair than the previous one, it hides the auction mechanism from the bidder

higher price, but there is no guarantee they can earn this profit. Additionally, these mechanisms allow for students to transfer points to one another by selling a low value course for a high value and use the market to swap seats in courses. This opens the gates for potential collusion and even the idea of a cartel which can persist from year to year through a point transfer is not farfetched. For these reasons, the Wharton auction is extremely interesting, but may not result in fair outcomes or equitable market access for all participants. The new system, which solicits weighted preferences across all courses and normalizes these against a budget for each student (4000 points for the first year, 5000 for the second), then internally conducts something like a mixture of a auction and a constrained optimization problem to determine each student's schedule. This system allows more robust expressions of preferences and, while complex, eliminates the strategic manipulation aspect of the auction and market sale of courses.

Overall, there is a wide variety across course registration auctions. While the ideal auction would be both easy for students to understand and participate in while both truthfully aggregating revealed preference data and allocating seats in courses in a fair manner, it is clear from some examples of these auctions that this is not always the case. Strategic bidding and inherent underlying inquities undermine these goals, which ultimately make designing such an auction very difficult.

In light of the variety of course registration auctions in use around the country, we consider the design and implementation of one at Colby College to replace the current system. Colby's registration system can be described as a sort of 'priority lottery' where students freely register for classes and are randomly given seats or pruned according to a different set of stratifications for each course. For example, if a course with 20 seats is listed as "Priority by

seniority", and there are 15 seniors and 15 juniors registered, all 15 seniors receive seats and five juniors are randomly selected. While this system is fair, it ignores variance among preferences and may award a seat to a student who is just looking for a fourth course to take over a student who needs that course for her major or has a passion for the subject material.

Students at Colby typically enroll in 16 credits per semester and a course is typically four credits, so most students take four courses per term. Colby's system allows registration for up to 20 credits without needing administrative approval, so it is possible to register for as many as five courses at a time. Where this becomes a problem is that it becomes a weakly dominant strategy for a student to register for an extra course, even if she doesn't plan on actually taking that class because there are no penalties to trying to get that extra seat. This artificially inflates demand for seats in courses and because there is no elicitation of preferences in the registration process, students with high preference for a course could be pruned and the seat be given to a student with no intention of taking the course.

At Colby, relatively few courses end up overenrolled, but those that do are consistent from term to term; introductory science courses, labs at popular times, intermediate 'bottleneck' courses[5] in popular majors, and upper-level economics electives are among those courses which often need to prune students. Because so few courses actually end up overenrolled, it is important for the auction system to recognize that students should only need to bid for one or two courses in each semester, as a zero-bid should be sufficient to secure a seat in the vast majority of classes.

[5] Courses like Microeconomic Theory or Research Methods in Psychology which are important prerequisites for upper-level courses.

Another important factor to consider when designing an auction for Colby is ease of understanding. While business schools can make some assumptions about background knowledge and experience of their students and therefore use complex mechanisms in their course auction implementations, an auction for Colby should be as simple and easy to understand as possible, as it must be used by undergraduate students from a wide range of disciplines with a wide range of knowledge.

The following auction design is based off of Harvard's Kennedy School of Government's course registration process. In this auction, students participate in an open registration process where they sign up for courses they wish to take. Following the close of this round, any student who has signed up for one or more overenrolled course receives a notification to participate in an auction round. In this round, students receive an endowment of points which are used to bid only on overenrolled courses. This allows students who are trying to get seats in more than one popular course to honestly express preferences among those courses. Since points are not bankable, the auction is resolved as a first-price one with seats going to the highest bidders, The waitlist for these classes is then ordered from highest to lowest among the unsuccessful bidders. After the resolution of the auction, there is an add/drop period which is first-come-first-served for open seats, with the exception of courses with existing waitlists.

One distinct advantage to this auction is that the open registration and add/drop portions are identical to the current system Colby uses, so it could be easily built upon existing infrastructure. Additionally, the system does not preclude giving priority to students by seniority, major, minor, or other status, as an affirmative action policy could be implemented to pad the bids of these students in formulaic ways, so that, for example, a course with priority by

seniority could pad bids of seniors by 20 points, juniors by 15, sophomores by ten and first years by five. It also no makes it more difficult for a student who registers for a fifth class with no intention of taking it to negatively impact a student with strong preferences for taking that course. Consider a situation where two students register for an overenrolled course. Student A has strong preferences for taking the class and Student B plans on dropping it if she gets into her other four choices, and the bidding endowment is 100 points with no affirmative action policies. If Student B only needs to bid on this course, she bids her full 100 points and then drops the class, which has the same outcome as if she had submitted a zero bid due to the prioritization of the waitlist. If Student B needs to bid on another course as well, she should bid all 100 points on that other course in accordance with her preferences. In either case, Student B's actions do not affect the ability of Student A to enroll in the class.

Overall, course auctions are a novel and efficient way to allocate the scarce resource of seats in college classes. While not many schools use them, there is a wide variety of implementations across the schools that do. In considering what makes a course auction good or bad, we must consider both how effectively seats are allocated and how easy it is to understand and participate in. For these reasons, an auction based on Harvard's Kennedy School of Government's registration process would be ideal for implementation at Colby College due to its simplicity and similarity to the existing system, as well as the hope that using such a system will correct some of the strategic flaws in the current course registration process, which can be very frustrating for students.

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